Technical and conceptual advances of LQG

Jerzy Lewandowski
Uniwersytet Warszawski

Challenges for QG, Rome, September 8-12, 2014

Three messages and one anecdote

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- Quantum geometry coupled to the scalar field: a new operator

$$\hat{q}^{ab}\hat{\phi}_{,a}\hat{\phi}_{,b}\mathrm{det}\hat{q}$$

a new application for LQG, neat, "cute", L, Sahlmann - 2014

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  Finally we have self adjoint scalar constraint operators

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- A witty anecdote about the maid and gardener - instructive, funny Kurt Vonnegut, Breakfast of Champions - 1973

The connection variables of LQG

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$$-2\mathrm{Tr}(\tau_i \tau_j) = \delta_{ij},$$

$$A^i_a = \Gamma^i_a + \gamma K^i_a, \qquad E^a_i = \frac{1}{16\pi G\gamma} e^j_b e^k_c \epsilon^{abc} \epsilon_{ijk}.$$

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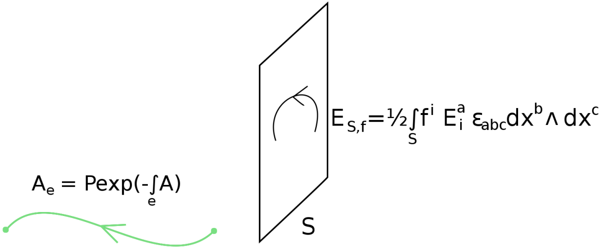

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$$E_{S,f} = \tfrac{1}{2}\int_S f^i\, E_i^a\, \varepsilon_{abc}\, dx^b \wedge dx^c$$

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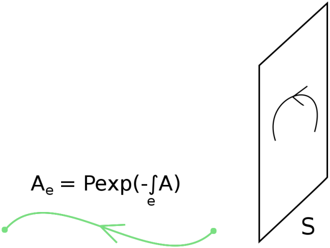

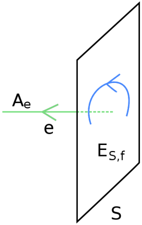

$$\{A_e, E_{S,f}\} = \tfrac{1}{2}\, A_e \cdot f$$

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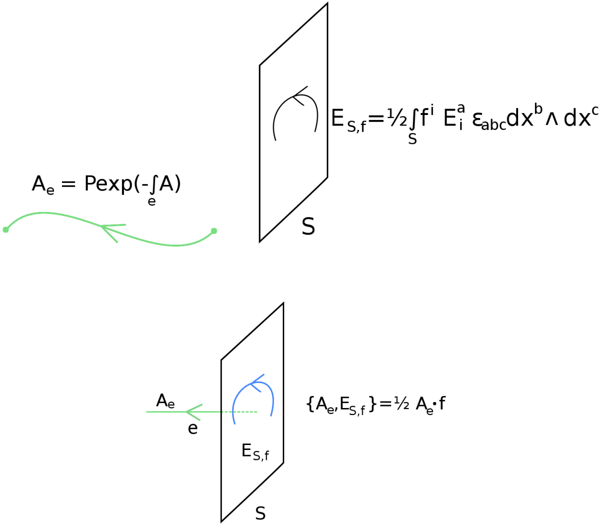

e and S ranges curves and, respectively, 2-surfaces in Σ.

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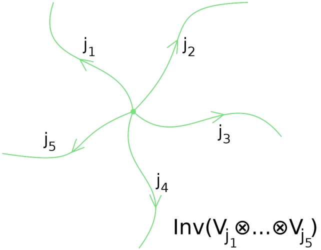

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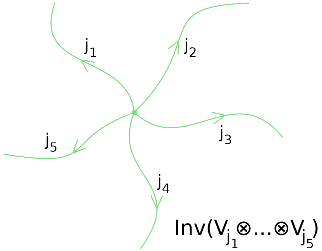

$$\Gamma \ni e_I \mapsto j_I, \quad \Gamma \ni v \mapsto \iota_v \in \mathrm{Inv}\, V_{j_1} \otimes \ \dots \ \otimes V_{j_k}$$

Spin-networks

$$U_g \Psi(A) = \Psi(g^{-1} A g + g^{-1} dg) \tag{2}$$
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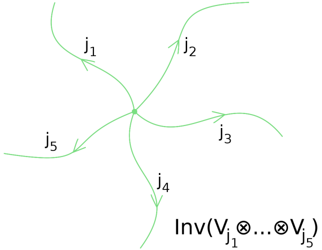

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$$|\Gamma, j, \iota > \text{ - a spin-network state}$$

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$$C_\phi(x) \;=\; (8\pi G\gamma)^{-\frac{3}{2}} \frac{\pi^2(x)}{\sqrt{|\det E|}} \;+\; \frac{1}{2}(8\pi G\gamma)^{\frac{7}{2}} \phi_{,a}\phi_{,b} E^a_i E^b_i \sqrt{|\det E|} \;+\; (8\pi G\gamma)^{\frac{3}{2}} V(\phi(x))\sqrt{|\det E|} \tag{4}$$

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Quantum:

$$|\varphi> : \;\; \pi \mapsto e^{-\frac{i}{\hbar}\int \pi\varphi} \qquad (5)$$

$$< \varphi \mid \varphi' > \;=\; \delta_{\varphi\varphi'} \qquad (6)$$

$$\hat{\phi}(x)|\varphi> = \; i\hbar\frac{\delta}{\delta\pi(x)}|\varphi> \;=\; \varphi(x)|\varphi>, \qquad (7)$$

$$e^{-\frac{i}{\hbar}\int \hat{\pi}\varphi}|\varphi'> \;=\; |\varphi + \varphi'> \qquad (8)$$

The new operator

L,Sahlmann 2014 (coming)

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Similarity with the QM on a graphs Rovelli, Vidotto 2008 and with the Ma operator Ma 2002

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provided the RHS is independent of refinements of the partition

$$\Sigma \;=\; \bigcup_k \Sigma_k.$$

The miracles of LQG

- The Hilbert space of states of quantum geometry

$$\mathcal{H}_{\rm kin}$$

with a Diff invariant $(\,\cdot\,|\,\cdot\,)$

- ∞-free derivations of operators of quantum geometry

$$\mathcal{O}(\hat{E})$$

- local frame rotation invariant states:

$$\Psi(A) \;= \Psi(g^{-1}Ag + g^{-1}dg)$$

- Averaging with respect to Diff, and the Hilbert space of Diff invariant states,

$$\mathcal{H}_{\rm Diff}$$

- ∞-free derivations of operators of quantum connection

$$\mathcal{O}(\hat{F})$$

in $\mathcal{H}_{\rm Diff}$ Rovelli,Smolin 1994, Ashtekar,L 1995, Thiemann 1998

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    the Master constraint program (Thiemann)

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L, Sahlmann 2014 (coming)

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$$\mathcal{O}(x) \;=\; \sqrt{\det E(x)},\; \sqrt{\det E}(x)\mathrm{Ric}(x),\; C_{\rm gr}(x)$$

The quantum scalar contraint

With this result we can:

- Consider $\hat{C}^\dagger(N)$ (it has sufficiently large domain).
- Define

$$\hat{C}(x)_{\rm sym} = \frac{1}{2}(\hat{C} + \hat{C}^\dagger)$$

- Find a self adjoint extension $\hat{C}_{\rm s.a.}$
- Spectrally expand each

$$\mathcal{H}_{\{x_1,\ldots,x_k\}} = \int^{\oplus} dc_1 \ldots dc_k \mathcal{H}^{c_1\ldots c_k}_{\{x_1,\ldots,x_k\}}$$

- promote $\mathcal{H}^{0\ldots 0}_{\{x_1,\ldots,x_k\}}$ to be solutions to the quantum scalar constraint

Solutions to the matter free LQG

The elements of $\mathcal{H}_{\{x_1,\ldots,x_k\}}$ have to be farther averaged with respect to the vertex non-preserving Diff(Σ) (the vector constraint),

$$\mathcal{H}_{\{x_1,\ldots,x_k\}}\ \eta_{\rm new}(\Psi)\ \mapsto\ \frac{1}{k}\sum_{[f]\in {\rm Diff}(\Sigma)/{\rm Diff}(\Sigma)}\eta_{\rm new}(U_f\Psi)$$

The map passes to the $\mathcal{H}^{0\ldots0}_{\{x_1,\ldots,x_k\}}$ space, and it's image defines a Hilbert space

$$\mathcal{H}_{(k)}$$

of k-vertex solutions to the scalar AND vector constraint.
The full Hilbert space is

$$\mathcal{H}_{\rm phys}\ =\ \oplus_k \mathcal{H}_{(k)}$$

Observables - emergence of points

Duch, Kaminski, L, Świezewski 2014, Bodendorfer, L, Świezewski 2014 (coming)

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Similarly for P^{ij} and Π.

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determined on the vector constraint surface

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Summary, outlook