INTRODUCTION TO QUANTUM REDUCED LOOP GRAVITY

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Conceptual and Technical Challanges for Quantum Gravity
Roma, 9th September 2014

Fixing the frame

QRLG can be seen as the gauge-fixed quantization of LQG, in the frame in which the metric tensor and the triads are both diagonal.

1) one can always fix a frame in which the three-metric tensor is diagonal

$$dl^2 = a_1^2(dx^1)^2 + a_2^2(dx^2)^2 + a_3^2(dx^3)^2$$

2) one can always the gauge in which the triads is diagonal

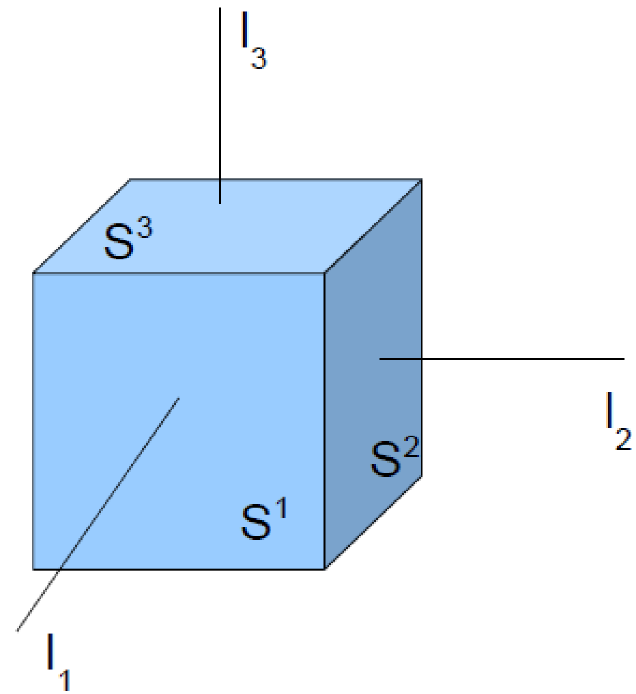
additional terms

$$E_i^a = p^i \delta_i^a \qquad A_a^i = c_i \delta_a^i + \dots$$

Let us implement 1) and 2) **weakly** in the gauge-invariant kinematical Hilbert space of LQG:

A little bit of notation:

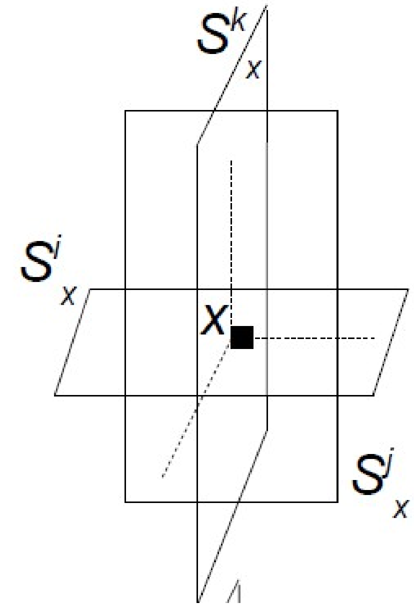
let us call i=1,2,3 the principal direction, l_i the associated links and S^i the dual surfaces



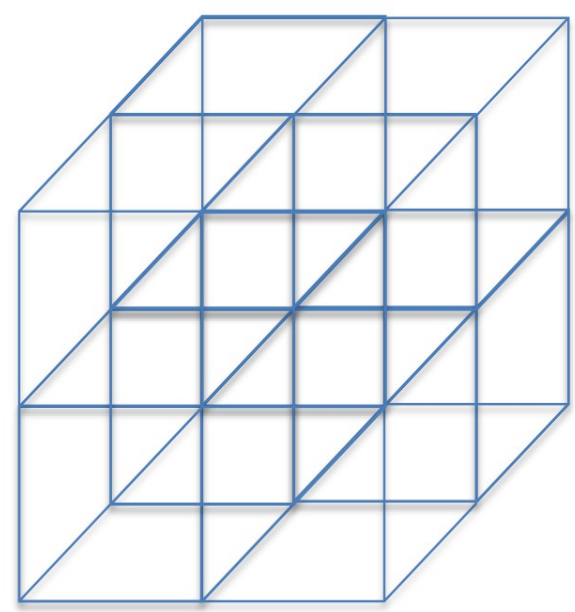
1) gauge-fixing condition in terms of fluxes

$$\eta_x^{km} = \delta^{ij} E_i(S_x^k) E_j(S_x^m) = 0, \qquad k \neq m, \quad \forall x \in \Sigma$$

$$\langle \psi | \eta_x^{km} | \phi \rangle = 0, \qquad k \neq m, \quad \forall x \in \Sigma$$



Restriction to reduced graphs.

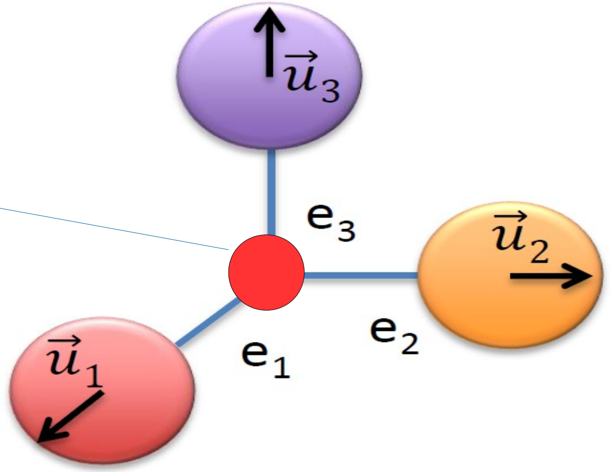


$$\langle \Gamma, \mathbf{j_l}, \mathbf{x_n} | \psi \rangle = \prod_{n \in \Gamma} \langle \mathbf{j_l}, \vec{\mathbf{u}}_\mathbf{l} | \mathbf{j_l}, x_n \rangle \prod_l \psi_l^{j_l, \vec{u}_l}$$

Reduced intertwiners

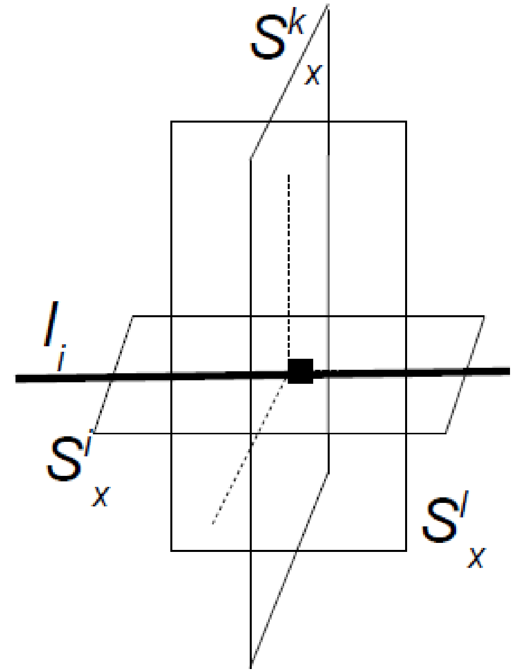
Alesci, FC, Rovelli

SU(2) intertwiner
projected on coherent states:
Reduced intertwiner



2) gauge-fixing condition in terms of fluxes

$$\chi_i(S) = \epsilon_{ij}{}^{k} E_k(S^j) = 0$$



We mimic the imposition of simplicity constraints in EPRL model: we impose strongly

$$\chi^2 = \sum_i \chi_i^2$$

Engle, Livine, Pereira, Rovelli

$$\langle h|\Gamma, \mathbf{j_l}, \mathbf{x_n}\rangle = \prod_{n\in\Gamma} \langle \mathbf{j_l}, x_n|\mathbf{j_l}, \vec{\mathbf{u}}_\mathbf{l}\rangle \prod_{l} {}^{l}D^{j_l}_{m_l m_l}(h_l)$$

Reduced intertwiners

$m_l = \pm j_l$

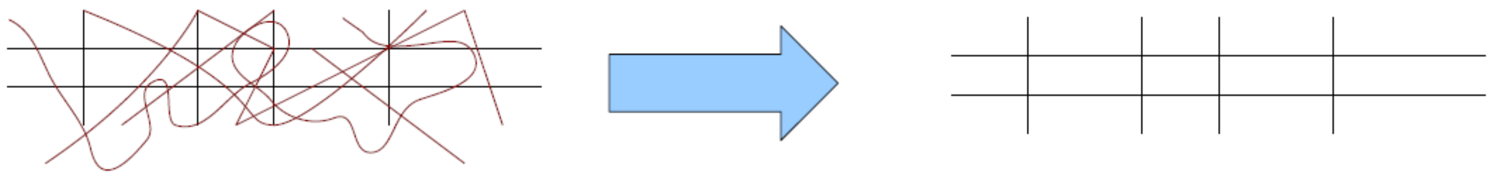
Fluxes acts diagonally

$$ {}^{R}\hat{E}_i(S^i)\, {}^{l}D^{j_l}_{m_l m_l}(h_l) = 8\pi\gamma l_P^2\, m_l\, {}^{l}D^{j_l}_{m_l m_l}(h_l) \qquad l = l_i \cap S^i \neq \oslash$$

To summarize: Quantum-reduced Hilbert space is obtained by

_ restricting to reduced graphs

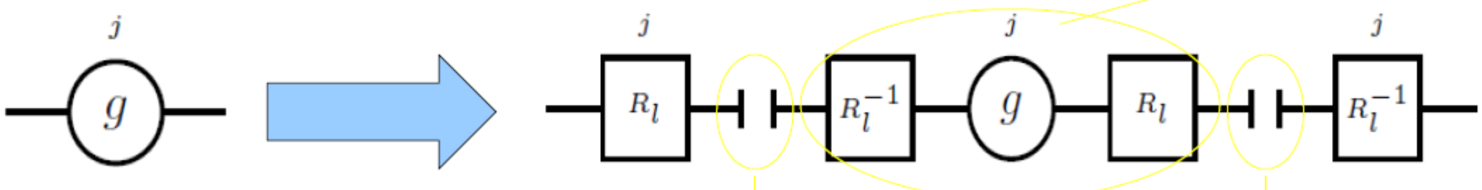
Reduced diffeomorphisms



_ projecting SU(2) representations and operators as follows

$^{i}D^{j}_{mn}(g)$

$D^{j}_{mn}(g) =$



Projection on maximum or minimum magnetic number

states: one or the other

operators: sum of the two

Fluxes $E_i(S^i)$ read the magnetic numbers of states based at links l_i

Scalar constraint

The scalar constraint can be defined by using the elements of the reduced Hilbert space:

ITS MATRIX ELEMENTS CAN BE <u>EXPLICITLY COMPUTED,</u> since the volume operator is diagonal

IT CAN BE <u>REGULARIZED</u>, thanks to reduced diffeo-invariance

Euclidean part:

$${}^R\hat{H}_E[N] = \sum_{\square} {}^R\hat{H}^m_{E\,\square}[N]$$

Cubulation

$${}^R\hat{H}^m_{E\,\square}[N] := N(\mathfrak{n})C(m)\;\epsilon^{ijk}\,\mathrm{Tr}\left[{}^R\hat{h}^{(m)}_{\alpha_{ij}}\,{}^R\hat{h}^{(m)-1}_{s_k}\left[{}^R\hat{h}^{(m)}_{s_k},\,{}^R\hat{V}\right]\right]$$

obtained by projecting SU(2) elements on reduced elements

diagonal operator

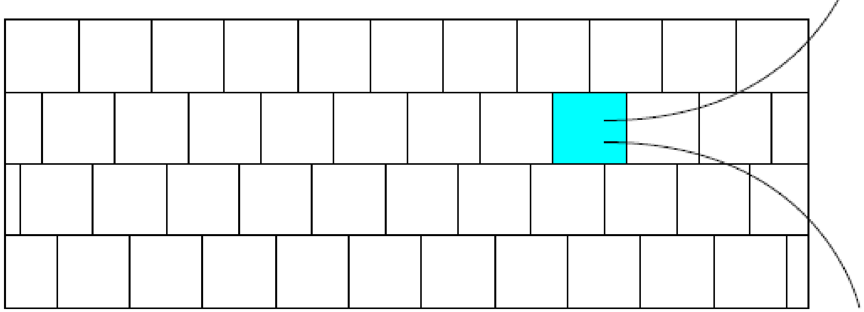
Let us evaluate the expectation value of the following operator on proper states:

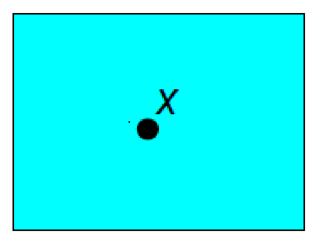
$$ {}^{R}\hat{H} = \frac{1}{\gamma^2}\, {}^{R}\hat{H}_E $$

which in the classical limit described the dynamics of a spacetime made of a collection of local Bianchi I patches (BKL conjecture)

$$ H[N] = \frac{1}{\gamma^2} \sum_x V(x) N(x) \left[\sqrt{\frac{p^1 p^2}{p^3}} c_1 c_2 + \sqrt{\frac{p^2 p^3}{p^1}} c_2 c_3 + \sqrt{\frac{p^3 p^1}{p^2}} c_3 c_1 \right](x) $$

Volume of local patch in x





homogeneous patch

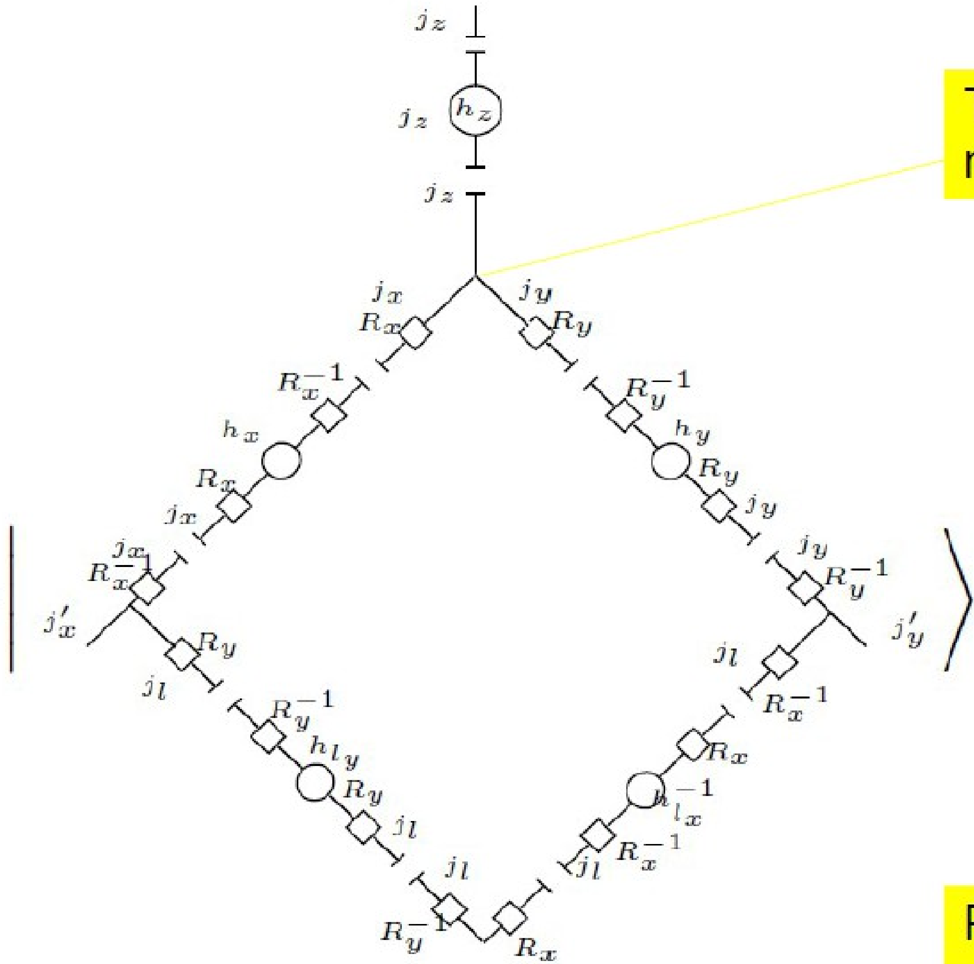
$$ dl^2 = a_1^2(x)(dx^1)^2 + a_2^2(x)(dx^2)^2 + a_3^2(x)(dx^3)^2 $$

$$ p^i = \frac{a_1 a_2 a_3}{a_i} \qquad c_i = \frac{\gamma}{N} \dot{a}_i $$

We describe each homogeneous patch via: states which already contains the loop added by one of the three terms into the Euclidean scalar constraint (dressed node):

Three-valence node

$$|\mathbf{n}^z\rangle_R = \Big|$$



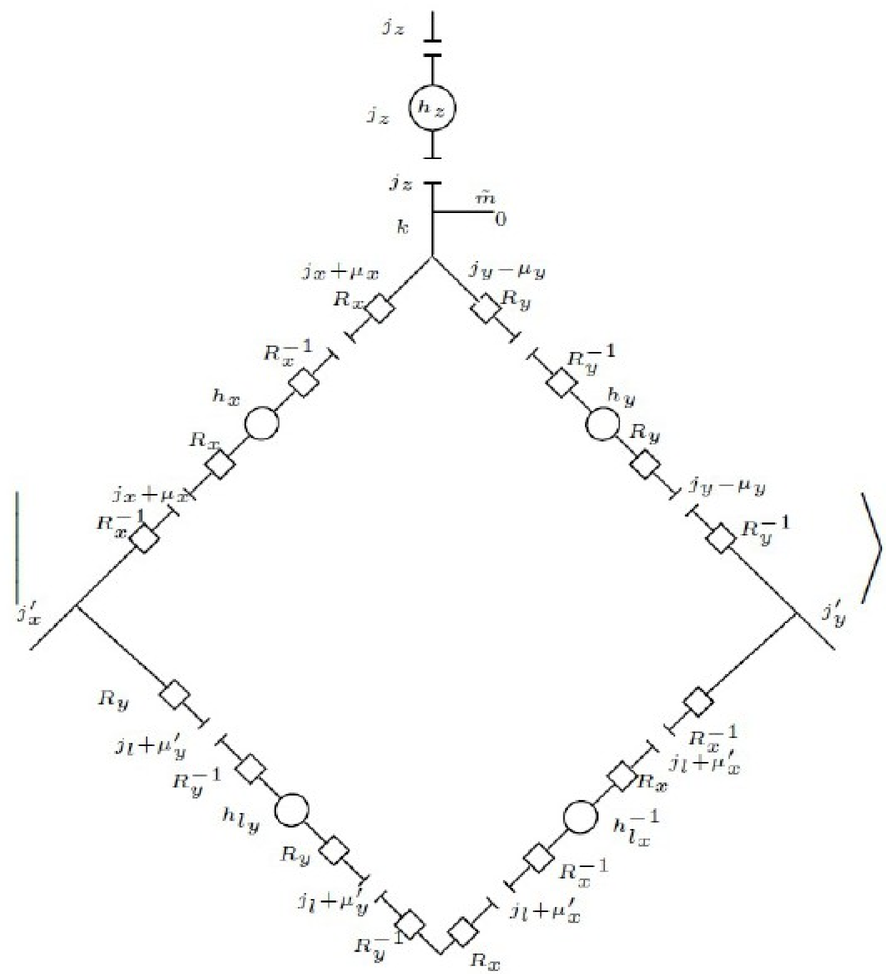
Projections on maximum magnetic numbers

Non-graph changing operator

The final result:

$$\mathrm{Tr}\left[{}^{R}\hat{h}^{(m)}_{\alpha_{[xy]}}\,{}^{R}\hat{h}^{(m)-1}_{s_z}\,{}^{R}\hat{V}\,{}^{R}\hat{h}^{(m)}_{s_z}\right]|\mathfrak{n}^z\rangle_R =$$

$$= \sum_{\mu'_x\mu'_y\mu_x,\mu_y=\pm m} H^{m\;j_x j'_x j_y j'_y}_{\mu_x\mu'_x\mu_y\mu'_y}(j_z, j_l)$$



$$H^{m\ j_x j'_x j_y j'_y}_{\mu_x \mu'_x \mu_y \mu'_y}(j_z, j_l) = (8\pi\gamma l_P^2)^{3/2} \sum_k \sum_{\tilde{m}} \sum_{\mu=\pm m} \sqrt{j_x\, j_y\, (j_z + \mu)}$$

$$s(\mu)\, C^{mm}_{mm\,\tilde{m}0} \left\{ \begin{matrix} k & \tilde{m} & j_z \\ j_y - \mu_y & m & j_y \\ j_x + \mu_x & m & j_x \end{matrix} \right\} \left\{ \begin{matrix} j'_x & j_l + \mu'_y & j_x + \mu_x \\ m & j_x & j_l \end{matrix} \right\} \left\{ \begin{matrix} j_l + \mu'_x & j'_y & j_y - \mu_y \\ j_y & m & j_l \end{matrix} \right\}$$

(Euclidean) scalar constraint matrix elements

TO BE CONTINUED....