New Developments in Three-Dimensional Gravity

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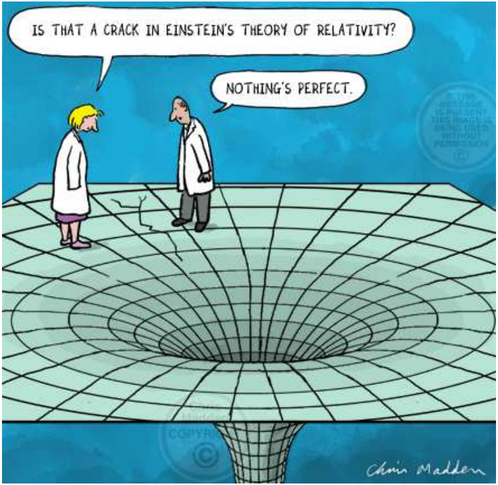
What is Massive Gravity ?

Massive Gravity is the name we have given to the attempt to understand what the gravitational force would be like if the graviton, the carrier of the gravitational force, has a small, but non-zero, mass

Massive Gravity might have cosmological applications (dark energy?)

Warning: it is not a simple thing to modify Einstein !



There are issues at both ends of the energy scale !

A New Motivation

D-dimensional gravity theories may describe interesting Conformal Field Theories in one dimension lower via the so-called AdS/CFT correspondence

There are interesting non-unitary conformal field theories

Example: chiral/critical gravity theories describe logarithmic conformal field theories

Why Three Dimensions ?

- Three dimensions is an interesting 'testing ground'

- 3D gravity is a 'Chern-Simons gauge theory'

Achúcarro, Townsend (1986), Witten (1988)

- Einstein-Cartan gravity in AdS_3 is dual to a CFT_2 with

$$c_{L/R} = \frac{2\ell}{2G}$$

D. Brown, M. Henneaux (1986)

3D Cosmological Gravity

Deser, Jackiw, 't Hooft (1984); Achúcarro, Townsend (1986); Witten (1988)

First-order formalism: Dreibein $e_\mu{}^a$ and spin-connection $\omega_\mu{}^a$

$\mu, a = 0, 1, 2$

$$S_{\text{cosmological}} = \frac{1}{16\pi G} \int e \cdot \left[d\omega + \frac{1}{2}\left(\omega \times \omega - \frac{1}{3}\Lambda\, e \times e\right)\right]$$

Counting Degrees of Freedom

- split into time and space components: $\mu = (0, i)$ with $i = 1, 2$
- we have $2 \times 2 \times 3 = 12$ phase space coordinates
- the Lagrange multipliers $(e_0{}^a, \omega_0{}^a)$ generate 6 first-class constraints
- we have $\frac{1}{2}(12 - 2 \times 6) = 0$ DoF

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Two Approaches

you can obtain a massive graviton by adding either

- explicit mass terms

or

- higher-derivative terms

Explicit Mass Terms


Fierz and Pauli (1939)

- the free massive graviton, with mass m, is a spin-2 particle described by a symmetric tensor field $h_{\mu\nu}(x)$

- for $m = 0$ this tensor can be viewed as the linearized approximation to a metric tensor $g_{\mu\nu}(x)$:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) + O(\kappa^2 h^2)$$

- the Fierz-Pauli kinetic term is the linearization of the Einstein-Hilbert term of general relativity

The Fierz-Pauli mass term

$$\mathcal{L}_{\text{FP}}(\text{mass}) \sim m^2 \left(h^{\mu\nu} h_{\mu\nu} - h^2\right) \qquad h \equiv \eta^{\mu\nu} h_{\mu\nu}$$

The Fierz-Pauli mass term

- breaks the linearized g.c.t. of the kinetic term:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

- contains a reference metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$

- requires fine-tuning ($\Rightarrow$ can be written as 'CS-like' $\sim \delta \cdot e \times e\,!$)

Non-linear Fierz-Pauli

First proposals for a non-linear mass term date back to

Salam, Strathdee (1969); Isham, Salam, Strathdee (1970); Zumino (1970)

There are many dangers on the road!

- 1970: van Dam, Veltman, Zakharov
- 1972: Vainshtein
- 1972: Boulware, Deser
- 2010: de Rham, Gabadadze, Tolley
- 2013: Deser, Waldron
- AdS/CFT ? See R. Davison (2013); Blake, Tong (2013); Blake, Tong, Vegh (2013)

Higher Derivatives

$$4\text{D} : \quad \mathcal{L} \sim R + a\left(R_{\mu\nu}{}^{ab}\right)^2 + b\left(R_{\mu\nu}\right)^2 + c\,R^2$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no (bulk) massless spin 2 !

Deser, Jackiw, 't Hooft (1984)

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

Fierz-Pauli and Higher Derivatives

- a higher-derivative theory can be constructed starting from the second-order derivative FP equations

  - $\left(\Box - m^2\right)\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \qquad \partial^\mu \tilde{h}_{\mu\nu} = 0$

  and solving for the differential subsidiary condition in terms of a symmetric tensor $h_{\mu\nu}$ as follows:

$$\tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta}\epsilon_\nu{}^{\gamma\delta}\partial_\alpha\partial_\gamma h_{\beta\delta} \equiv G^{\text{lin}}_{\mu\nu}(h)$$

- this solution restores the (linearized) general coordinate transformations of general relativity !

New Massive Gravity

Substituting the solution back into the FP equations leads to the following fourth-order derivative equations:

$$(\Box - m^2)\ G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization: $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right] \; : \text{ "New Massive Gravity"}$$

- invariant under diffeomorphisms
- requires fine-tuning
- 'wrong' sign in front of EH term

A 'Chern-Simons-like' Reformulation

$$\mathcal{L}_{\rm NMG}(e,\omega;f,h) = M_P\Big\{ -\sigma e\cdot R(\omega) + \Lambda_0\, e\cdot e\times e + + h\cdot T - \frac{1}{m^2}\Big(f\cdot R(\omega) + \frac{1}{2} e\cdot f\times f\Big)\Big\}$$

$$T = de + \omega\times\omega \; : \qquad \text{torsion tensor}$$

- Note that $\Lambda_0 \neq \Lambda$
- reduces to Einstein-Hilbert plus Fierz-Pauli in linearized approximation

An Issue

Li, Song, Strominger (2008)

- New Massive Gravity has 'good' massive gravitons but 'bad' BTZ black holes

$$m_{\mathrm{FP}} = -m^2\big(\sigma - \frac{\Lambda}{2m^2}\big)\,, \qquad c_{L/R} = +\frac{3\ell}{2G}\big(\sigma - \frac{\Lambda}{2m^2}\big)$$

Note: 'chiral' or 'critical' graviy for $\Lambda_{\mathrm{crit}} = 2m^2\sigma$

- Ultimately, this is due to the 'wrong' sign in front of the Einstein-Hilbert term

- we are therefore interested in models in which we can have a different sign in front of the Einstein-Hilbert term

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CS-like Models of 3D Gravity

introduce N 'flavours' of Lorentz-vector valued 1-form fields

$$V^{ra}\,,\, a = 0, 1, 2;\, r = 1, 2, \ldots, N$$

$$L_{\mathrm{CSL}} = \frac{1}{2} g_{rs} V^r \cdot dV^s + \frac{1}{6} f_{rst} V^r \cdot V^s \times V^t$$

- g_{rs} is a constant invertable flavour-space metric
- f_{rst} is a constant totally symmetric flavour tensor

These CS-like models allow for a very general analysis !

We first restrict to Models with at most 4 Derivatives

N	1-forms	$S(g)$
2	$\{e, \omega\}$	CEH
3	$\{e, \omega; h\}$	TMG
4	$\{e, \omega; f, h\}$	NMG

Topologically Massive Gravity

Deser, Jackiw, Templeton (1982)

$$S_{\mathrm{TMG}}[g] \;=\; \int d^3x\,\sqrt{-\det g}\,(\sigma R - 2\Lambda_0) + \frac{1}{2\mu}\int \mathrm{tr}\left\{\Gamma d\Gamma + \frac{1}{3}\Gamma^3\right\}$$

$$\frac{1}{\mu}C_{\mu\nu} + \sigma G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = 0 \quad \text{with } C_{\mu\nu} \quad \text{Cotton tensor}$$

For critical value of Λ_0 a non-unitary 'logarithmic' boundary CFT arises

Li, Song, Strominger (2008); S. Carlip, S. Deser, A. Waldron and D. K. Wise (2008); Grumiller, Johansson (2008)

CS-Like Reformulation

Carlip (2008)

$$L_{TMG}[e,\omega,h] = -\sigma e\cdot R(\omega) + \frac{\Lambda_0}{6} e\cdot e\times e + h\cdot T(\omega) + \frac{1}{2\mu}\left(\omega\cdot d\omega + \frac{1}{3}\omega\cdot\omega\times\omega\right)$$

with

$$T(\omega) = de + \omega\times e\,, \qquad R(\omega) = d\omega + \frac{1}{2}\omega\times\omega$$

Minimal Massive Gravity

N	1-forms	$S(g)$	no $S(g)$
2	$\{e, \omega\}$	CEH	
3	$\{e, \omega; h\}$	TMG	MMG
4	$\{e, \omega; f, h\}$	NMG	

Minimal Massive Gravity

Hohm, Merbis, Routh, Townsend + E.B. (2014)

$$\frac{1}{\mu} C_{\mu\nu} + \bar{\sigma} G_{\mu\nu} + \bar{\Lambda}_0 g_{\mu\nu} = -\frac{\gamma}{\mu^2} J_{\mu\nu}$$

with

$$\begin{aligned} J_{\mu\nu} &= \frac{1}{2\det g}\, \varepsilon_\mu{}^{\rho\sigma} \varepsilon_\nu{}^{\tau\eta} S_{\rho\tau} S_{\sigma\eta} \\ &= R_\mu{}^\rho R_{\rho\nu} - \frac{3}{4} R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R^{\rho\sigma} R_{\rho\sigma} - \frac{5}{8} R^2 \right) \end{aligned}$$

$$D_\mu J^{\mu\nu} = 0 \quad \text{on-shell!}$$

the J tensor does not contribute to linearisation about a Minkowski vacuum

CS-Like Reformulation

$$L_{MMG} = L_{TMG} + \frac{\alpha}{2} e \cdot h \times h$$

- for $\mu \to \infty$ L_{MMG} reduces to L_{NC}
- for finite μ the clash between gravitons and black holes can be avoided
- matter couplings are possible

Arvanitakis, Routh, Townsend (2014)

Zwei Dreibein Gravity

N	1-forms	$S(g)$	no $S(g)$
2	$\{e, \omega\}$	CEH	
3	$\{e, \omega; h\}$	TMG	MMG
4	$\{e, \omega; f, h\}$	NMG	ZDG

Zwei-Dreibein Gravity

we introduce two ('zwei') Dreibeine e_1, e_2 and two independent spin-connections ω_1, ω_2

Hassan, Rosen (2012); Hinterbichler, Rosen (2012)

$$L_{\text{ZDG}}(e_1, \omega_1; e_2, \omega_2) \;=\; \sigma\, L_{\text{NC}}(e_1, \omega_1) \;+\; L_{\text{NC}}(e_2, \omega_2) \;+\; L_{\text{int}}(e_1, e_2)$$

with interaction term

$$L_{\text{int}}(e_1, e_2) \quad \sim \quad \beta_1 e_1 \cdot e_1 \times e_2 \;+\; \beta_2 e_1 \cdot e_2 \times e_2$$

Assumption : only $e \equiv \beta_1 e_1 + \beta_2 e_2$ is is invertable !

special case: $\beta_2 = 0 \;\Rightarrow\; e^a = e_1^a$

de Haan, Hohm, Merbis, Townsend + E.B. (2013); Bañados, Deffayet, Pino (2013)

Generalization to Higher Derivatives

N	1-forms	$S(g)$	no $S(g)$
2	$\{e, \omega\}$	EH	
3	$\{e, \omega; f\}$	TMG	MMG
4	$\{e, \omega; f, h\}$	NMG	ZDG
6	$\{e, \omega; f_1, h_1; f_2, h_2\}$	'cubic extended' NMG	DDG
$\vdots$			
N	$\{V_I\}, \ I = 1, \ldots, N$	'EMG'	'VDG'

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Summary and outlook

I have shown how to introduce a number of interesting 'ghost-free' models of gravity in 3D

Some of them avoid the clash between gravitons and black holes

They should be studied further in the context of AdS/CFT

What can we learn from these 3D models about gravity in 4D ?

Thank you for your Attention !