# CHALLENGING STRING THEORY: GRAVITY vs GAUGE

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# Foreword

After 46 years, there is NO experimental evidence for **String Theory** except for ...

# Gravity and Electro-magnetism

Indeed, String Theory predicts

- **massless vector** in the **open string** spectrum
- **massless tensor** in the **closed string** spectrum

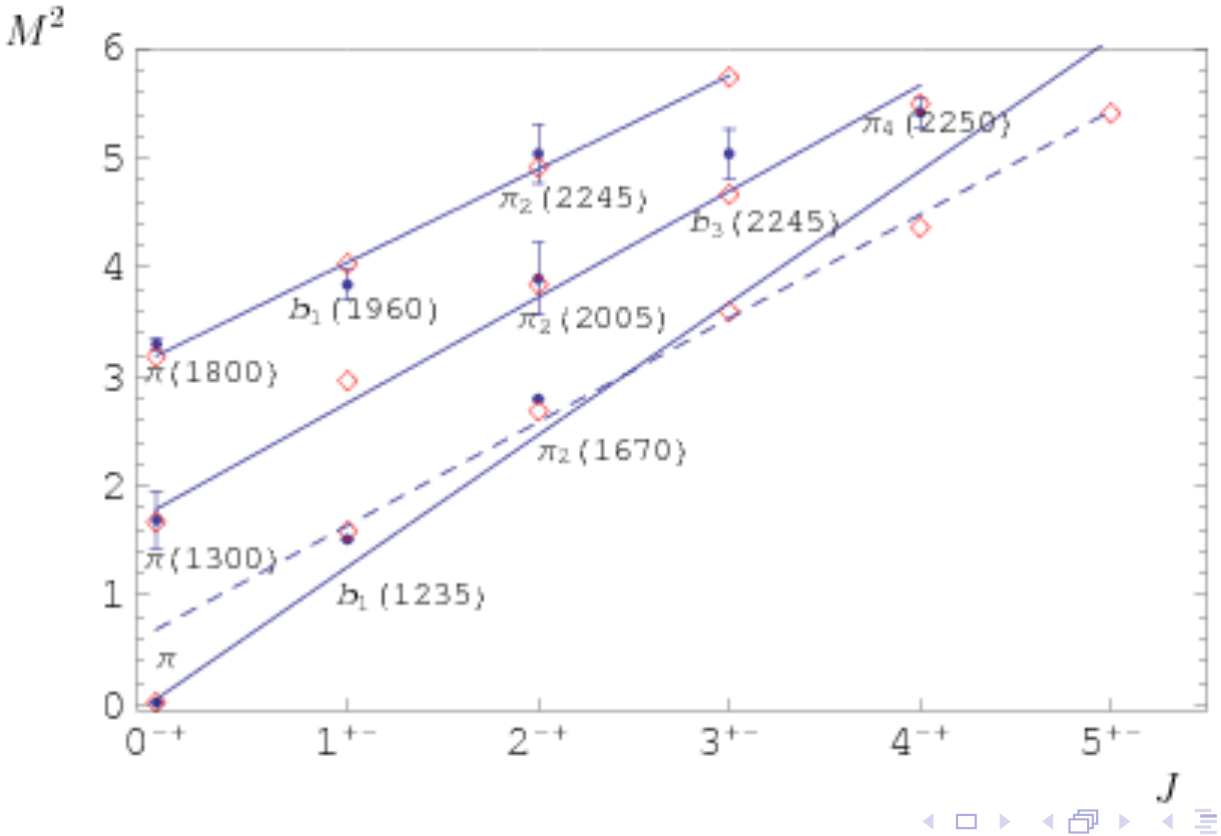
But String Theory also predicts

- Extra massless fields: dilaton, anti-symmetric tensors (axions), .... moduli
- Infinite tower of massive higher spin particles
- Definite but 'wrong' number of space-time dimensions:
  - **26** for **Bosonic Strings**
  - **10** for **Superstrings**
- Supersymmetry

# Plan

- Intro
  - (Non) planar duality
  - From hadrons to the Planck scale
  - GSO, SUSY and G-S
- $Gravity = (Gauge)^2$
  - KLT relations
  - MHV and all that
  - Is $\mathcal{N} = 8$ SuGra finite?
- $Gauge = \sqrt{Gravity}$
  - Open and unoriented strings
  - D-branes, M-branes and M-theory
- $Gauge \approx Gravity$
  - Holography
  - (non)AdS/(non)CFT correspondence
  - Fluxes, non-perturbative effects
- Outlook

# Once upon a time ...



# Open strings

Veneziano amplitude

$$\mathcal{A}(s,t) = g_s^2 \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' s)}{\Gamma(1-\alpha' s-\alpha' t)}$$

Planar duality ... open strings

$$\mathcal{A}(s,t) = \mathcal{A}(t,s)$$

Massless vector but scalar tachyon (Higgs-Englert-Brout boson?) and ... infinite tower of massive higher spin 'mesons'

$$T_s = 1/\alpha' \approx 1 GeV^2$$

Hagedorn transition $d(M) \approx \exp \sqrt{\alpha'} M$, $T_H \approx \sqrt{T_s}$ [Hagedorn; Hardy-Ramanujan]

# Closed Strings

Shapiro-Virasoro amplitude, non planar duality

$$\mathcal{A}(s,t,u) = g_s^2 \frac{\Gamma(1-\alpha'\frac{s}{4})\Gamma(1-\alpha'\frac{t}{4})\Gamma(1-\alpha'\frac{u}{4})}{\Gamma(1+\alpha'\frac{s+t}{4})\Gamma(1+\alpha'\frac{t+u}{4})\Gamma(1+\alpha'\frac{u+s}{4})}$$

Massless tensor boson = graviton ???
Soft UV behaviour, finite Quantum Gravity + ...
Fundamental strings [Scherk, Schwarz]

$$T_s = 1/\alpha' \approx M^2_{Planck}$$

# Super Strings

Supersymmetry on the world-sheet [Neveu-Schwarz, Ramond]

$$\delta X^\mu = \epsilon \Psi^\mu \quad , \quad \delta \Psi^\mu = \epsilon \partial X^\mu$$

G-S-O projection [Gliozzi, Olive, Scherk] $\longrightarrow$ Space-time supersymmetry [Brink, Green, Schwarz]

$$\theta_3^4(0|\tau) - \theta_4^4(0|\tau) - \theta_2^4(0|\tau) = 0$$

*Aequatio identica satis abstrusa* [Jacobi]
Covariant BRST Quantization and 'topological' perturbative expansion [Polyakov; Friedan, Martinec, Shenker; ...]

# Green-Schwarz mechanism and Heterotic Strings

- Exactly 30 years ago [Green, Schwarz]
  Anomaly cancellation in Type I strings (un-oriented open and closed) for SO(32) gauge group

$$\delta_{YM} C^{R-R}_{\mu\nu} = Tr(\alpha_{YM} F_{\mu\nu}) + ... \qquad \delta_{YM} A_\mu = D_\mu \alpha_{YM}$$

- Heterotic String $L \neq R$ [Gross, Harvey, Martinec and Rohm]
  Left-moving superstring $\otimes$ Right-moving bosonic strings
  26-10 $=$ 16 rank of SO(32) or E(8)$\times$ E(8)

# Compactifications

- Calabi-Yau three-folds with $SU(3)$ holonomy [Candelas, Horowitz, Strominger and Witten] $\longrightarrow$ chiral $\mathcal{N}=1$ supersymmetric models with $E(6)$ gauge group ('standard embedding') and $N_{gen}=\mathcal{X}_{CY}/2$
- 1985-95 Heterotic Model Building, non-standard embedding $\longrightarrow$ SO(10), SU(5), ..., SU(3)×SU(2)×U(1), with $N_{gen}\neq\mathcal{X}_{CY}/2$
- Winding states, T-duality: large volume $\sim$ small volume

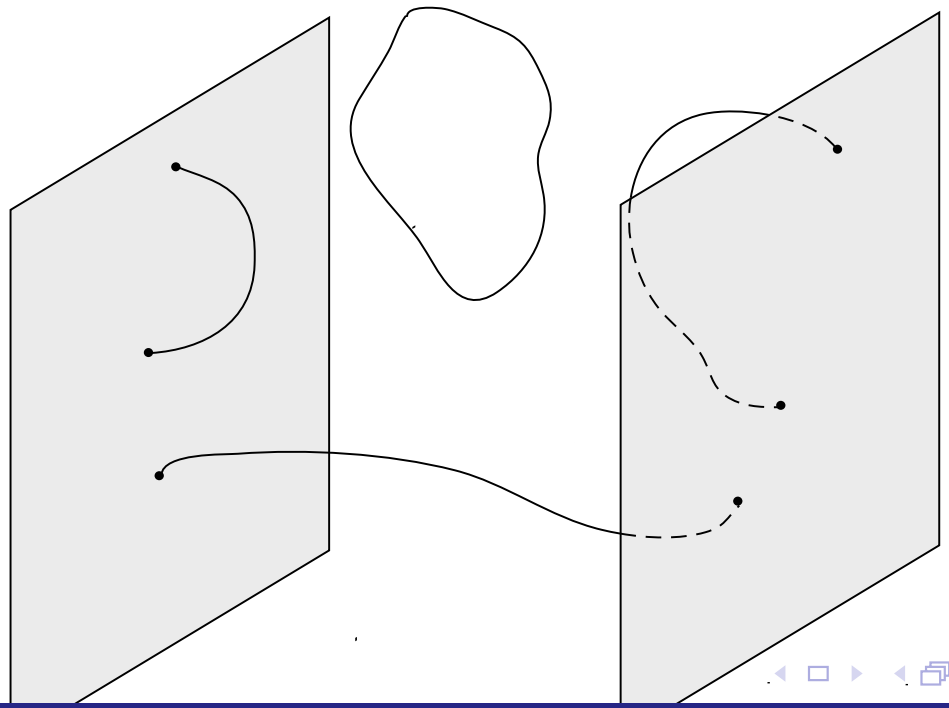
$$R\sim\alpha'/R$$

Mirror symmetry, quantum geometry, non-geometric compactifications

- Modification of uncertainty principle [Amati, Ciafaloni, Veneziano]

$$\Delta p\Delta x>\frac{\hbar}{2}\left(1+\frac{\Delta x^2}{\alpha'}\right)$$

$$Gravity = (Gauge)^2$$



# (Super)gravity from (super)strings

At low energies $\alpha' E^2 << 1$, effective action for the massless fields

- Closed strings (sphere + torus + ...)

$$\mathcal{L}_{eff} = e^{-2\phi}[R + (\partial\phi)^2 + H_3^2 + ... + \alpha' R^2 + ... + \alpha'^3 R^4 + ...]$$

$$+[\alpha'^3 R^4 + ...] + ...$$

- Open strings (disk + annulus + ...)

$$\mathcal{L}_{eff} = (e^{-\phi} + 1 + ...)[F^2 + \alpha' F^3 + \alpha'^2 F^4 + ...] + ...$$

# Closed strings from open strings

Graviton vertex operator $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$V_h(k) = h_{\mu\nu} \partial_z X^\mu \bar{\partial}_z X^\nu e^{ikX}$$

BRST invariance: $k^2 = 0$, $k^\mu h_{\mu\nu} = 0$, $\eta^{\mu\nu} h_{\nu\mu} = 0$
Gauge boson vertex operator

$$V_a(k) = a_\mu \partial_t X^\mu e^{ikX}$$

BRST invariance: $k^2 = 0$, $k^\mu a_\mu = 0$
Gravity = (Gauge)$^2$

$$V_h(k) = V_a^L(k/2) \otimes V_a^R(k/2)$$

# Scattering Amplitudes and KLT relations [Kawai, Lewellen, Tye]

Open superstring 4-point amplitude for vector bosons (disk)

$$\mathcal{A}_4(s,t) = -\frac{1}{2} g_s^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' s)}{\Gamma(1-\alpha' s-\alpha' t)} a_1^{\mu_1} a_2^{\mu_2} a_3^{\mu_3} a_4^{\mu_4} K_{\mu_1\mu_2\mu_3\mu_4}(k_i)$$

$$= \mathcal{A}_4^{SYM}(s,t) \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1+\alpha' u)}$$

Closed superstring 4-point amplitude for gravitons (sphere)

$$\mathcal{M}_4 = \kappa^2 \sin(\pi\alpha' t/4) \frac{\Gamma(-\alpha' s/4)\Gamma(-\alpha' t/4)}{\Gamma(1-\alpha' s/4-\alpha' t/4)} \frac{\Gamma(-\alpha' t/4)\Gamma(-\alpha' u/4)}{\Gamma(1-\alpha' t/4-\alpha' u/4)}$$

$$h_1^{\mu_1\nu_1} h_2^{\mu_2\nu_2} h_3^{\mu_3\nu_3} h_4^{\mu_4\nu_4} K_{\mu_1\mu_2\mu_3\mu_4}(k_i/2) K_{\mu_1\mu_2\mu_3\mu_4}(k_i/2)$$

$$= \frac{4\kappa^2}{g_s^4} \sin(\pi\alpha' t/4) \mathcal{A}_4(s/4,t/4) \mathcal{A}_4(t/4,u/4)|_{h_i = a_i \otimes a_i}$$

At tree level generalises to $n$ points, $(n-3)!$ terms in the sum

At loop level ... research project: looking for good PhD students!

# MHV and all that

In $D = 4$, helicity spinors $k^\mu \gamma_\mu u(k) = 0$, $k_{\alpha\dot\alpha} = u_\alpha \bar u_{\dot\alpha}$, $k^2 = 0$
Contractions $\langle uv\rangle = -\langle vu\rangle = \varepsilon^{\alpha\beta} u_\alpha v_\beta$, $[\bar u \bar v] = -[\bar v \bar u] = \varepsilon^{\dot\alpha\dot\beta} \bar u_{\dot\alpha} \bar v_{\dot\beta}$
Circular polarisations

$$a^+_\mu(k) = \bar v(q)\gamma_\mu u(k) \quad a^-_\mu(k) = \bar u(k)\gamma_\mu v(q)$$

At tree level, Yang-Mills is effectively (maximally) SUSY

$$\mathcal{A}_n(+,+,...,+) = 0 \quad \mathcal{A}_n(-,+,...,+) = 0$$

Maximally Helicity Violating amplitudes [Parke,Taylor]

$$\mathcal{A}_n^{MHV}(+,+,...,-_i,...,-_j,...,+) = \frac{\langle u_i u_j\rangle^4}{\langle u_1 u_2\rangle ... \langle u_n u_1\rangle}$$

# Generating Tree Amplitudes

Using 'effective' supersymmetry, on-shell $\mathcal{N}=4$ super-field

$$\Phi = a^+ + \lambda^+_A \eta^A + \frac{1}{2}\varphi_{AB}\eta^A\eta^B + \frac{1}{3!}\lambda^-_{ABC}\eta^A\eta^B\eta^C + a^-\eta^1\eta^2\eta^3\eta^4$$

Generating function of (super)MHV amplitudes in ($\mathcal{N}=4$ super)YM [Nair, Witten]

$$\mathcal{F}^{MHV}_{n,SYM}(u_i,\eta_i) = \frac{\mathcal{A}^{MHV}_n}{\langle u_i u_j\rangle^4}\delta^8(\sum_i u_i^\alpha \eta_i^A)$$

Generalized to nextMHV, next-nextMHV, ... next$^k$MHV both at tree level and loops

# (Super)gravity MHV Amplitudes

Doubling of $\eta's$ $h^{+2} = a^+ a^+$, $\psi^{+3/2} = a^+ \lambda^+ \oplus \lambda^+ a^+$, ...

Using field-theory limit of KLT relations

$$\mathcal{M}_4^{MHV} = s_{14} \mathcal{A}_4^{MHV}(1,2,3,4) \mathcal{A}_4^{MHV}(1,3,2,4)$$

$$\mathcal{M}_5^{MHV} = s_{12} s_{34} \mathcal{A}_5^{MHV}(1,2,3,4,5) \mathcal{A}_5^{MHV}(2,1,4,3,5)$$
$$+ s_{13} s_{24} \mathcal{A}_5^{MHV}(1,3,4,2,5) \mathcal{A}_5^{MHV}(3,1,2,4,5)$$

etc, get ($\mathcal{N} = 8$ super) gravity MHV amplitudes at tree level [MB, Elvang, Freedman]

$$\mathcal{G}_{n,sgrav}^{MHV}(u_i, \hat{\eta}_i) = \frac{\mathcal{M}_n^{MHV}}{\langle u_i u_j \rangle^8} \delta^{16}(\sum_i u_i^\alpha \hat{\eta}_i^a)$$

Generalized to nextMHV, next-nextMHV, ... next$^k$MHV at tree level and loops ... [Brandhuber, Heslop, Travaglini, Sokatchev, Eden, Arkani-Hamed, ....]

# On the UV finiteness of $\mathcal{N} = 8$ supergravity

Using generalised unitarity cuts, $\mathcal{N} = 8$ supergravity UV finite beyond original expectations [Bern, Dixon, Dunbar, Kosower]

Assuming global continuous non-linearly realised $E7(7)$ symmetry NO $D^4R^4$, $D^6R^4$ counter-terms [Kallosh]

In fact NO $E7(7)$ invariant counter-term below 7-loops !!!

... I am willing to bet[1]

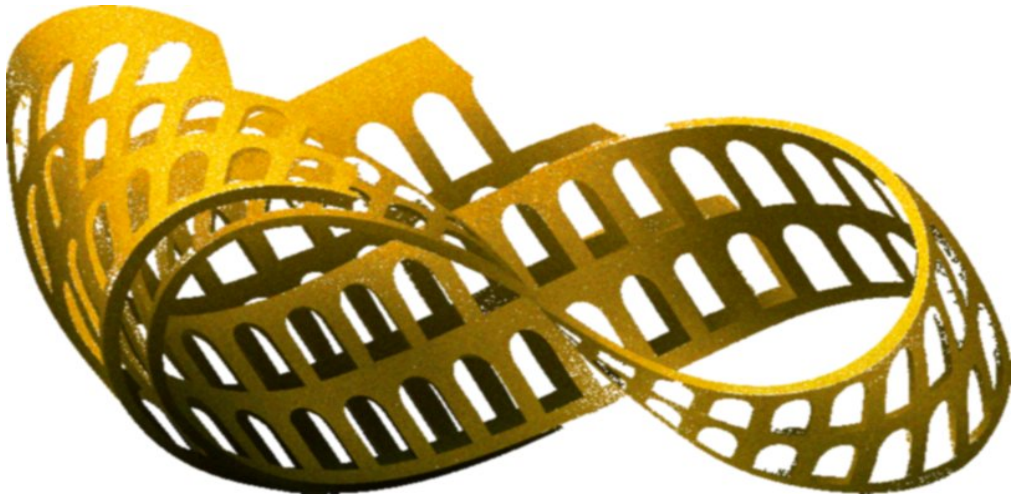
If $\mathcal{N} = 8$ supergravity were UV finite, where are the non-perturbative BPS states?

Large black holes $A_H \neq 0$ OK [MB, Ferrara, Kallosh]

1/2 BPS states $A_H = 0$ singular in $D = 4$, non-singular up-lift to $D > 5$ KK modes, present in Type II super-strings, non-decoupling [Green, Ooguri, Schwarz]

[1] You may find it useful the classification of $D^8R^4$ counter-terms and beyond at 7,8,9 loops [Beisert, Elvang, Freedman, Kiermaier].

$$Gauge = \sqrt{Gravity}$$



# Open strings from closed strings

Despite some 'phenomenological' success of Heterotic CY compactifications

- Many undetermined parameters (moduli fields)
- Hard [but not impossible] to get $N_{gen} = 3$
- Several chiral exotics or vector-like pairs
- Problems with Grand-Unification [$gauge \approx gravity$]

$$M_{Pl}^2 = \frac{M_s^8 V_{comp}}{g_s^2} \quad , \quad g_{GUT}^2 = \frac{g_s^2}{M_s^6 V_{comp}} \approx \frac{1}{25}$$

Yet, for perturbative closed strings systematic procedure

- Conformal Invariance
- Modular Invariance

# Type I and other Type II *un-Orientifolds*

- Unoriented open and closed strings from L-R symmetric oriented closed strings (Type II OK, Heterotic $L \neq R$ KO):
  - Gravity, dilaton and 2-form + susy from closed strings

$$\mathcal{T} = \frac{1}{2}\sum_i n_{ij}\mathcal{X}_i(\tau)\overline{\mathcal{X}_j(\tau)} \quad , \quad \mathcal{K} = \frac{1}{2}\sum_i k_i \mathcal{X}_i(2it)$$

  - Vector bosons and gaugini from open strings

$$\mathcal{A} = \frac{1}{2}\sum_{i,a,b} a_i^{ab} N_a N_b \mathcal{X}_i(it/2) \quad , \quad \mathcal{M} = \frac{1}{2}\sum_{i,a} m_i^a N_a \hat{\mathcal{X}}_i((1+it)/2)$$

- Transverse channel: tadpole cancellation $[\widetilde{\mathcal{K}} + \widetilde{\mathcal{A}} + \widetilde{\mathcal{M}}]_{RR_0} = 0$
- Boundary CFT's $T(z) = \overline{T(z)}$, $J(z) = \pm\overline{J(z)}$, ...
- 1986-96 Type I Systematics *'without'* D-branes and $\Omega$-planes

[Sagnotti, MB, Pradisi, ... Stanev, ... Angelantonj, ... Morales, ....]

# Pre D-brane phenomenology

- Non-supersymmetric (non) tachyonic models in $D \leq 10$
- Vertex operators for R-R potentials, 'minimal coupling'
- Rank reduction with $B_{NS-NS} \neq 0$, 'non-commutative' Wilson lines
- Several tensor mltps in $D = 6$, generalized GS mechanism [ ... Ferrara, Riccioni]
- $\mathcal{N} = 1$ susy chiral models in $D = 4$ with $N_{gen} = 3$
- Magnetic fields and SUSY breaking

# D-branes

- Open strings terminate on D-branes [Dai, Leigh, Polchinski]

$$\partial_n X^\mu = 0 \quad (Neumann, longitudinal)$$

$$X^i = x_0^i \quad (Dirichlet, transverse)$$

- D-branes carry Ramond-Ramond charge [Polchinski; MB, Pradisi, Sagnotti]

$$T_{Dp} = |Q_{Dp}| = \frac{\omega_p}{g_s(\alpha')^{\frac{p+1}{2}}}$$

- Low-energy dynamics: SYM in $D = p + 1$ with $U(N)$ gauge group [Polchinski, Witten]
- $U(N) \to SO(N)/Sp(N)$ from 'mirror-like' $\Omega$-planes $T_{\Omega p} < 0$

# Other p-branes

1/2 BPS solitonic p-branes charged wrt different anti-symmetric tensor fields

- NS5-brane couples to $\tilde{B}_6$ magnetic dual to $B_2$ (sourced by F1) [Callan, Harvey, Strominger]
- Tensions: $T_{F1} \approx 1/\alpha'$, $T_{NS5} \approx 1/g_s^2(\alpha')^3$, ... [... Bergshoeff, Marrani, Riccioni, Pradisi]
- String dualities: T, S (strong/weak), ... U
- p-brane democracy, ... M-theory [Hull, Townsend; Witten]
- (BPS/extremal) BH's as bound states of strings and branes. Exact micro-state counting [Strominger, Vafa; Callan, Maldacena]
- M(atrix)-theory proposal [Banks, Fischler, Shenker, Susskind]

# Un-oriented D-brane worlds

Large Extra Dimension low string tension [Antoniadis, Arkani-Hamed, Dimopoulos, Dvali] $M_s \approx TeV$ and $g_s << 1$ (no need for GUT)

$$M_{Pl} \approx \frac{M_s^4 \sqrt{V_{int}}}{g_s} \quad , \quad g_{YM}^2 = \frac{g_s}{V_{p-3} M_s^{\frac{p-3}{2}}}$$

(N-MS)SM on intersecting / magnetized / fractional D-branes

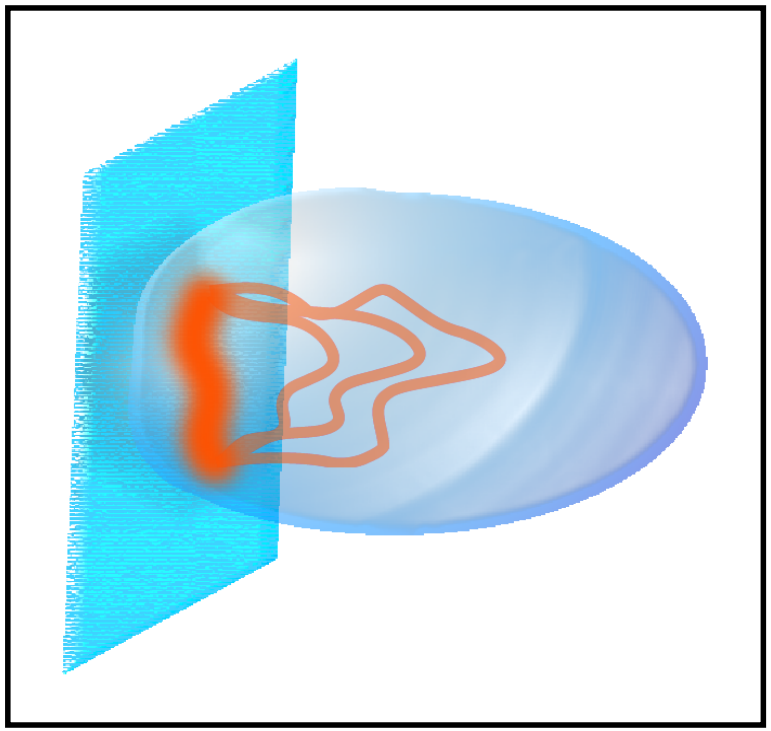
[Ibanez, Quevedo, Marchesano, Uranga, Cvetic, Shiu, Blumenhagen, Lüst, Stieberger, Kiritsis, Schellekens, ...]

Gravity in 10-d bulk: weak from dilution

$$M_{Pl,(4)}^2 = M_{Pl,(D)}^{D-2} L_{int}^{D-4}$$

Spectacular signatures at LHC from Z' (extra U(1)'s), KK excitations, higher spins, (small) BH's [Perelstein, Peskin; Dudas, ...; Chialva, Iengo; MB, Santini; Anchordequoi, Lüst, Stieberger, Taylor; ...]

# Gravity $\approx$ Gauge



# AdS/CFT correspondence

Holographic duality between gravity / strings in anti De Sitter space and (super)conformal theory on the boundary $z \approx 0$

$$\mathcal{Z}_{AdS}[\Phi \approx z^{d-\Delta} J(x)] = \langle \exp \int J(x) \mathcal{O}_\Delta(x) dx \rangle_{CFT}$$

For scalars [Witten; Gubser, Klebanov, Polyakov; ... Breitenlohner, Freedman]

$$M^2 L^2_{AdS} = \Delta(\Delta - d) \geq -\frac{d^2}{4}$$

Holographic principle ['t Hooft, Susskind]
Gravity as an emergent phenomenon [Maldacena, ...]

# Extremal Black-branes and black-holes

Reissner-Nordström black-hole in $D = 4$

$$ds^2 = -\left(1 - \frac{2G_N M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \frac{dr^2}{1 - \frac{2G_N M}{r} + \frac{Q^2}{r^2}} + r^2[d\vartheta^2 + \sin\vartheta^2 d\varphi^2]$$

Extremal $|Q| = G_N M$, smooth everywhere.
String theory, *e.g.* D3-branes wrapping 3-cycle in CY $A_H = 4\pi Q^2$
or D0-D2-D4-D6 bound-state $A_H = 4\pi\sqrt{Q_0 Q_2 Q_4 Q_6}$
Near horizon $r \approx |Q|$, $u = Q^2/r - |Q|$

$$ds^2 = Q^2[\frac{-dt^2 + du^2}{u^2} + (d\vartheta^2 + \sin\vartheta^2 d\varphi^2)]$$

$AdS_2 \times S^2$ Bertotti-Robinson geometry

# Extremal D3-branes

Metric

$$ds^2_{4+6} = H^{-1/2} dx\cdot dx + H^{1/2} dy dy$$

$H = 1 + \frac{L^4}{r^4}$, $r^2 = y \cdot y$, $\nabla^2_y H = 0$
Constant dilaton, $F_5 = (1 + \star) d^4x \wedge dH$
Near horizon $r \approx 0$, get $AdS_5 \times S^5$

$$ds^2 \approx \frac{r^2}{L^2} dx\cdot dx + \frac{L^2}{r^2} dr^2 + L^2 ds^2_{S^5} = \frac{L^2}{z^2}(dx\cdot dx + dz^2) + L^2 ds^2_{S^5}$$

Infinite red-shift, decoupling of low-energy modes from the rest

# Maldacena conjecture

Type IIB superstrings in $AdS_5 \times S^5$ with $\int_{S^5} F_5 = N$

*holographically dual to*

$\mathcal{N} = 4$ SYM theory in $D = 4$ with gauge group $SU(N)$

$$4\pi g_s = g_{YM}^2 \quad , \quad L^4 = \alpha'^2 g_{YM}^2 N$$

Large $N$ planar limit, 'classical' strings (sphere)
Large radius $L$ (small curvature) strong 't Hooft coupling

# The harmonic oscillator of QFT

$\mathcal{N} = 4$ SYM theory $\{A^a_\mu, \lambda^a_{A\alpha}, \varphi^a_i\}$

- All fields in Adj, interactions $g_{YM} f_{abc}$
- $\beta(g_{YM}) = 0$ exactly, UV finite SCFT
- $\theta_{YM}$ dependence, exact $SU(4)$ R-symmetry
- Exact S-duality in the Coulomb phase
- Holographic duality: Green functions, ...
- Anomalous dimensions $\Delta = \Delta_0 + \gamma$
- OPE coefficients
- Integrability in the planar limit $N \to \infty$

# Further developments

- $AdS_3/CFT_2$ from D1/D5, $AdS_7/CFT_6$ from M5, $AdS_4/CFT_3$ from M2
- Domain wall, RG flows and Holographic Renormalization
- $\theta_{YM}$ dependence, instantons vs D-instantons $\rho = z$
- Small radius, Higher Spin Holography and *La Grande Bouffe*
- Finite temperature and density: $\eta/s = 1/4\pi$ ... RHIC
- Loops, IR divergences, exponentiation
- Amplitude - Wilson loop - Correlator triality $W(C) \sim e^{-T_s A_C}$
- Holographic QCD, QG plasma ... Kerr/CFT, ... AdS/CMT
- Warped and Flux compactifications 'Generalized Geometry'

# Fluxes and brane instantons

Internal fluxes generates a (super)potential [Gukov, Vafa, Witten]

$$W_{flux}(\mathcal{Z},\mathcal{S}) = \int_{CY} \Omega_3(\mathcal{Z}) \wedge (F_3^{RR} + \mathcal{S} H_3^{NS-NS})$$

Can stabilise several moduli fields

Need non-perturbative effects from Euclidean branes

Two classes of D-brane instantons

- 'Gauge' instanton $F = \pm\tilde{F}$, $\Sigma_{Dp}^{int} = \Sigma_{ED(p-4)}$ , ADHM for free!

$$W_{gauge} = \frac{M_s^{\beta} e^{-T(\Sigma)}}{\Phi^{\beta-3}}$$

- 'Exotic' instanton $F \neq \pm\tilde{F}$, $\Sigma_{Dp}^{int} \neq \Sigma_{EDp'}$ , no ADHM

$$W_{exotic} = M_s^{3-n} \Phi^n e^{-T(\Sigma)}$$

# Last but not least

Gravity from string scattering (off D-brane bound-states) at ultra high energies

- Fixed angle regime $\alpha' s \approx \alpha' |t| >> 1$, enhanced higher spin symmetry [Gross, Mende; MB, Lopez, Richter; Ko, Lee, Yang; Kao, Chan]
- Small angle $\alpha' s >> \alpha' |t|$, Regge regime, eikonal approximation [Amati, Ciafaloni, Veneziano; D'Appollonio, Di Vecchia, Russo, Veneziano; MB, Teresi; Black, Monni]

Higher derivative string scale corrections, loop corrections
Replace 'classical' concepts (area/entropy) with 'quantum' observables (cross sections, rates)
Near horizon ... another research project for PhD students

# Outlook

- Conceptual challenges
  - Black Holes and Singularities?
  - Holography beyond AdS?
  - Landscape or single vacuum?
  - Off-shell?
- Technical challenges
  - Quantization in the presence of (R-R) fluxes
  - Perturbative expansion: pure spinor vs NSR
  - Embedding SM (after SUSY breaking etc)
  - Inflation and origin of Dark Energy
  - Higher Spins and Hagedorn transition